IMPLICIT FINITE DIFFERENCE APPROXIMATION OF
THE 2D PRESSURE EQUATION

When we discretize the following:

\[
\beta \left( \frac{\partial P}{\partial t} \right) = \frac{2}{\Delta x} \left[ k_x \left( \frac{\partial^2 P}{\partial x^2} \right) \right] + \frac{2}{\Delta y} \left[ k_y \left( \frac{\partial^2 P}{\partial y^2} \right) \right]
\]

We get an implicit finite difference scheme.

If we then rearrange and take all the unknown terms to the LHS and the known terms to the RHS, we get an equation that has a set of five non-zero terms per grid block (these are the coefficients).

So in this 2D case it is known as a pentadiagonal matrix.

The set of linear equations are as follows:

\[
(a_{i-1,j} \times P_{i-1,j}^{n+1}) + (a_{i,j-1} \times P_{i,j-1}^{n+1}) + (a_{i,j} \times P_{i,j}^{n+1}) + (a_{i,j+1} \times P_{i,j+1}^{n+1})
\]

\[
+ (a_{i,j+1} \times P_{i,j+1}^{n+1}) = b_{i,j}
\]