IMPLICIT FINITE DIFFERENCE APPROXIMATION OF THE LINEAR PRESSURE EQUATION

Consider the spatial derivative $\left(\frac{\partial^2 P}{\partial x^2}\right)$ and take the derivative at the $(n+1)$ time level – the "unknown" level.

Time derivative is the same ∴

$$\left(\frac{\partial P}{\partial t}\right)_i \approx \frac{P_i^{n+1} - P_i^{n}}{\Delta t}$$

but the spatial derivative becomes:

$$\left(\frac{\partial^2 P}{\partial x^2}\right)_i \approx \frac{P_{i+1}^{n+1} + P_{i-1}^{n+1} - 2P_i^{n+1}}{\Delta x^2}$$

⇒ Equate the numerical finite difference approximations of each of the above derivatives (as required by PDE):

$$\frac{P_i^{n+1} - P_i^{n}}{\Delta t} \approx \frac{P_{i+1}^{n+1} + P_{i-1}^{n+1} - 2P_i^{n+1}}{\Delta x^2}$$

This equation cannot be arranged to give a simple expression for pressure.
It appears there are three unknowns at each grid point. To solve for this we use a set of <u>linear equations</u>

For this simple 1D PDE, it is clear that the matrix arising from the implicit finite difference method is <u>tridiagonal</u> – it has a maximum of three non-zero elements in any row and these are symmetric around the central diagonal.