IMPLICIT FINITE DIFFERENCE APPROXIMATION OF
THE LINEAR PRESSURE EQUATION

Consider the spatial derivative \((\frac{\partial^2 p}{\partial x^2})\)
and take the derivative of the \((n+1)\)
time level - the "unknown" level.

Time derivative is the same:

\[
\left( \frac{\partial p}{\partial t} \right)_i \approx \frac{P_i^{n+1} - P_i^n}{\Delta t}
\]

but the spatial derivative becomes:

\[
\left( \frac{\partial^2 p}{\partial x^2} \right)_i \approx \frac{P_{i+1}^{n+1} + P_{i-1}^{n+1} - 2P_i^{n+1}}{\Delta x^2}
\]

\[=\] Equate the numerical finite difference approximations of each of the above derivatives (as required by PDE):

\[
\frac{P_i^{n+1} - P_i^n}{\Delta t} \approx \frac{P_{i+1}^{n+1} + P_{i-1}^{n+1} - 2P_i^{n+1}}{\Delta x^2}
\]

This equation cannot be arranged to give a simple expression for pressure. It appears there are three unknowns at each grid point. To solve for this we use a set of linear equations.

For this simple 1D PDE, it is clear that the matrix arising from the implicit finite difference method is tridiagonal - it has a maximum of three non-zero elements in any row and these are symmetric around the central diagonal.