Finite difference approximation of the second derivative, $\left(\frac{\partial^2 P}{\partial x^2}\right)$

The definition of second derivative at $\left(\frac{\partial P}{\partial x}\right)_{x_i}$ is the rate of change of slope $(dP/dx)$ at $x_i$. Therefore, we can evaluate this derivative between $x_{i-1}$ and $x_i$ (bd approx.) and do the same between $x_i$ and $x_{i+1}$ (fd approx.) and take the rate of change between these two quantities with respect to $x$:

$$\left(\frac{\partial^2 P}{\partial x^2}\right) \approx \frac{\left(\frac{dP}{dx}\right)_{ifd} - \left(\frac{dP}{dx}\right)_{ibd}}{\Delta x}$$

$$\approx \frac{\left(\frac{P_{i+1} - P_i}{\Delta x}\right)_{ifd} - \left(\frac{P_i - P_{i-1}}{\Delta x}\right)_{ibd}}{\Delta x}$$

$$\therefore \left(\frac{\partial^2 P}{\partial x^2}\right) \approx \frac{(P_{i+1} + P_{i-1} - 2P_i)}{\Delta x^2}$$